Interregnum: Metaformal Realism and the Ontological Problematic

(Chapter 5 of Draft MS: *The Logic of Being: Heidegger, Truth, and Time*)

"Not empiricism and yet realism in philosophy, that is the hardest thing." -Wittgenstein

"A human is that being which prefers to represent itself within finitude, whose sign is death, rather than knowing itself to be entirely traversed and encircled by the omnipresence of infinity." -Badiou

I

In his 1951 Gibbs lecture, "Some basic theorems on the foundations of mathematics and their philosophical implications," drawing out some of the "philosophical consequences" of his two incompleteness theorems and related results, Kurt Gödel outlines a disjunctive alternative which, as I shall argue here, captures in a precise way the most significant consequences of contemporary formal reflection in considering the relationship of formalism to the real of being:

> Either mathematics is incompletable in [the] sense that its evident axioms can never be comprised in a finite rule, i.e. to say the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable Diophantine problems of the type specified […] (where the case that both terms of the disjunction are true is not excluded so that there are, strictly speaking, three alternatives).[1]

A consequence of this aporeatic ontological situation, as I shall try to show, is that the longstanding philosophical debate over the relative priority of thought and being that finds expression in discussions of "realism" and "anti-realism" (whether of idealist, positivist, or conventionalist forms) can only be assayed from the position of what I shall call a *meta-formal* reflection on the relationship of the *forms* of thought to the *real* of being. This is exactly the kind of reflection exemplified by Gödel's argument in the Gibbs lecture. Moreover, if Gödel's argument is correct, and if it bears (as I shall try to show it does) not only on the question of "mathematical reality" narrowly conceived but, more generally, on the very "relationship" of thought and being that is at issue in these discussions, it is also not neutral on this question of relative priority. Rather, it motivates a particular kind of *realism* – what I shall call "metaformal" realism -- that differs markedly both from "metaphysical realism" and from the varieties of "speculative realism" on offer today.

The type of realism I shall defend here is not a realism *about* any particular class or type of objects or entities. Thus it is not, *a fortiori*, an *empirical* or "ontic" realism or a naturalism (although I also do not

[1] Gödel (1951), p. 134.

think it is *inconsistent* with positions that march under these banners). In particular, its primary source is not any empirical experience but rather the experience of formalization, both insofar as this experience points to the real-impossible point of the *actual relation* of thinkable forms to being and insofar as it schematizes, in results such as Gödel's, the intrinsic capacity of formalization problematically to capture and decompose its *own* limits. As such a position on the form of the possible relationship of thought to being, it is (as I shall argue here) relevant to the "ontological" problematic of the possible thought of being "itself", and is even requisite for a furtherance of this problematic on realist terms today. In particular, because meta-formal realism is not an ontic realism about any particular domain of entities, but rather unfolds the inherent structural forms of thought in relation to the sense of "being", it can support an *ontological* realism that is appropriate to the formal indication of the "relationship" of ontological difference between being and beings and also to the formally indicated problematic of the underlying structure of time as it is "given" in relation to "being as such."

The most important yield of this ontological realism is the correlative possibility of a *temporal* realism that clarifies the structure of given time without reference to any subjective or anthropological origin for it in individuals, practices, languages or cultures. Whether or not one agrees with the exegetical claim that Heidegger himself, after *Being and Time,* increasingly seeks to distance himself from and repudiate a residual "anthropologism" or humanism that may still be suggested or implied in the "preparatory" fundamental analysis of Dasein (a repudiation that appears to find expression, for instance, in the forceful terms of the 1946 "Letter on Humanism"), the sort of realism that I argue for here is at least a *possible* position relevant to the ontological problematic, both as developed in *Being and Time* and as it yields Heidegger's later interrogation of the "truth" of Being independently of any relation to entities.[2] It also provides a concrete formal basis for critical arguments and positions that are unmistakably Heidegger's own. For on one hand, as I shall argue, the attitude or position of meta-formal realism as I shall develop it here provides a formal basis for the critique of any position that puts the *representing subject* at the basis of the possible thought of being by indicating the formal-ontological configuration that first underlies the ontological possibility of there being anything like a subject to begin with. On the other hand, and on the same realist terms, it provides a concrete basis for the critique of the identification of being with effective *actuality* [*Wirklichkeit*], that Heidegger sees as characteristic of the contemporary culmination of metaphysical thought and practice in the regime of technology and totalizing "enframing" [*Gestell*].

One decisive basis for the kind of realism I defend here is the chain of consequences following from the Cantorian event and the problematic accessibility of the infinite to mathematical thought, up to and including Gödel's incompleteness results. In particular, I shall argue, these consequences offer to challenge the traditional conception of the human as an essentially *finite* agent of *representative* thought by locating the implications of a point at which thought touches on a "real" that is impossible for such an agent. Through contemporary metalogic, this "real" is indicated, not only as a positive structure of excess, but also as a point of impasse: the point at which a finitely constructed formalism reflexively captures its own specific structural limits, as well as those of the finitely constituted capacities it underlies. Basing itself on this, the metaformal realism I shall develop more fully here

[2] GA 9.

might be formulated precisely, referring in passing to the Lacanian motto according to which "the Real can be inscribed only on the basis of an impasse of formalization," as a realism *of* the "Real" in something like Lacan's sense.[3] The "Real," in this sense, is thought as one of the three "registers" of psychological development; here it is both an inherent limit-point of structure and an obscurely constitutive underside for both of the other two "registers" of the Imaginary and the Symbolic. As such, it structurally articulates the subject's necessarily displaced or "barred" position in relation to what Lacan characterizes as the "thing" or the "object small a".[4] Thought in this way, the "real" is to be sharply distinguished both from "reality" in the sense of actuality and from any realm, regime or domain of actually existing objects. But as Lacan himself occasionally suggests, the problematic of "access" to the Real, at the structurally necessary point of the symbolic impasse which is, for him, formally constitutive of the very structure of the subject in the order of the symbolic, is structurally related to the "ontological" problematic of the "place" of being as such in relation to the factical life and structured language of the beings we are. As I shall argue here, this problematic, first developed (in *Being and Time*) as that of the constitutive structure of the kind of entity – Dasein – that is ontic-ontological in its constitutive relationship to being itself, and later (after the mid-1930s) as that of the truth of being that comes to light as time, both suggests and requires the rigorously formal realism that I shall defend on partially independent grounds.

To arrive at the disjunctive conclusion he draws in the lecture, Gödel draws centrally on a concept central to twentieth-century inquiry into the foundations of mathematics, that of a "finite procedure." Such a procedure is one that can be carried out in finite number of steps by a system governed by well-defined and finitely stateable rules, a so-called "formal system." As Gödel points out, there are several rigorous ways to define such a system, but they have all been shown to be equivalent to the definition given by Turing of a certain specifiable type of machine (what has come to be called a "Turing machine").[5] The significance of the investigation of formal systems for research into the structure of mathematical cognition and reality lies in the possibility it presents of rigorously posing general questions about the capacities of such systems to solve mathematical problems or prove mathematical truths. For instance, one can pose as rigorous questions i) the question whether such a system is capable of proving *all* arithmetic truths about whole numbers; and ii) whether such a system is capable of proving a statement of its own consistency. Notoriously, Gödel's first and second incompleteness theorems, respectively, answer these two questions, for *any* consistent formal system capable of formulating the truths of arithmetic, in the negative: given any such system, it is possible to formulate an arithmetic sentence which can (intuitively) be seen to be true but cannot be proven by the system,

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[3] Lacan (1973), p. 93.

[4] Lacan's concept of the "Real" is complex and undergoes many changes of specification and inflection over the course of his career. I do not take a view here about how precisely to define it or which formulation is most important, but seek only to preserve the link that is constitutive for Lacan between the Real and formalization at the latter's point of inherent impasse. For a very exhaustive and illuminating treatment of Lacan's concept, see Eyers (2012). I also discuss Lacan's motto and Badiou's reversal of it into his own claim for a "theory of the pass of the real, in the breach opened up by formalization…" in Livingston (2012), pp. 188-192.

[5] This is a formulation of the "Church-Turing" thesis, which holds that the structure of a Turing machine (or any of several provably equivalent formulations) captures the 'intuitive' notion of solvability or effective computability.

and it is impossible for the system to prove a statement of its own consistency (unless it is in fact *inconsistent*).

Gödel's argument from these results to his "disjunctive conclusion" in the lecture is relatively straightforward. The first incompleteness theorem shows that, for any formal system of the specified sort, it is possible to generate a particular sentence which we can "see" to be true (on the assumption of the system's consistency) but which the system itself cannot prove.[6] Mathematics is thus, from the perspective of any specific formal system, "inexhaustible" in the sense that no such formal system will ever capture *all* the actual mathematical truths. Of course, given any such system and its unprovable truth, it is possible to specify a *new* system in which that truth is provable; but then the new system will have its own unprovable Gödel sentence, and so on. The question now arises whether or not there is some formal system which can prove *all* the statements that *we* can successively see to be true in this intuitional way. If *not*, then human mathematical cognition, in perceiving the truth of the successive Gödel sentences, essentially *exceeds* the capacities of all formal systems, and mechanism (the claim that human mathematical coginition is, or is capturable by, a formal system) is false; this is the first alternative of Gödel's disjunction. If *so*, however, then there *is* some formal system that captures the capacities of human mathematical thought. It remains, however, that there will be statements that are undecidable for this system, including the statement of *its* consistency, which is itself simply an arithmetical statement. Thus it is impossible, on this alternative, simultaneously to identify the underlying principles on which actual mathematical cognition is based and to claim that these principles are both consistent and capable of deciding all mathematical problems. In this case there are thus classes of problems that cannot be solved by any formal method we can show to be consistent *or* by any application of our powers of mathematical cognition themselves; there are well-defined problems which will remain unsolvable, now and for all time.

We can further specify the underlying issue, and move closer to its philosophical significance, by noting that, by Gödel's second theorem, the undecidable Gödel sentence for each system is equivalent (even within the system) to a statement, within that system, of its own consistency. As Gödel emphasizes, it is (given classical assumptions) an implication of the *correctness* of any system of axioms that we might adopt for the purposes of arithmetic demonstration that the system be consistent; but then it is an implication of the second incompleteness theorem that if we are in fact *using* a specific (and consistent) formal system to derive all the mathematical truths (that we know) we could not know that we are. For if we could know this, i.e. if we could know the truth of the assertion of the consistency of the system, we would thereby know a mathematical truth that cannot be derived from that system. Accordingly, as Gödel says, it is

[6] I here state the first theorem, roughly and intuitively, appealing to a notion of "truth" that is in some ways problematic. For discussion of the issues involved in the difference between this and other, less potentially problematic statements, see Livingston (2012), chapter 6.

> …impossible that someone should set up a certain well defined system of axioms and rules and consistently make the following assertion about it: All of the axioms and rules I perceive (with mathematical certitude) to be correct and moreover I believe they contain all of mathematics.[7]

Thus if a system is (knowably) consistent it is, by that token, *demonstrably* incomplete; if it is complete, we cannot know it to be consistent (and hence we cannot know it to be correct). Accordingly, on the assumption that we are in fact using a finite procedure to demonstrate mathematical truths, the assumption of the consistency of the system we are actually using is shown to be *essentially* unsecurable in any way that is itself consistent with our (in fact) using (only) that system at all.

Again, by considering the question of the axiomatization of mathematics, we can see how the issue is connected to the problem of the accessibility of the infinite, and the higher levels of infinity. Specifically, in order to axiomatize arithmetic set-theoretically without contradiction, it is necessary to introduce axioms in a step-by-step manner, and in fact, as Gödel suggests, this process can be continued infinitely: thus

> Instead of ending up with a finite number of axioms, as in geometry, one is faced with an infinite series of axioms, which can be extended further and further, without any end being visible and, apparently, without any possibility of comprising all these axioms in a finite rule producing them.[8]

The successive introduction of the various levels of axioms corresponds to the axiomatization of sets of various order types; in each case the introduction of a new level of axioms corresponds to the assumption of the existence of a set formed as the *limit* of the iteration of a well-defined operation.

But each axiom "entails the solution of certain Diophantine problems, which had been undecidable on the basis of the preceding axioms;" in particular, according to a result that Gödel had achieved in the 1930s, the consistency statement for any given system of axioms can be shown to be equivalent to a statement asserting the existence of integral solutions for a particular polynomial. Instead of ending up with a finite number of axioms, as in geometry, one is faced with an infinite series of axioms, which can be extended further and further, without any end being visible and, apparently, without any possibility of comprising all these axioms in a finite rule producing them.[9]

Since consistency is undecidable within the system itself, so is the problem of the truth-value of the statement concerned, but it becomes decided in a stronger system which adds, as a new axiom, a statement of the former system's consistency (or something equivalent to this).[10] But since the problem

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[7] Gödel (1951), p. 133.
[8] Gödel (1951), p. 130.
[9] Gödel (1951), p. 130.
[10] The result that Gödel refers to in 1951 is that the consistency statement is equivalent to some statement of the form:
$\forall x_1 \ldots x_n \, \exists y_1 \ldots y_m \, [p(x_1, \ldots, x_n, y_1, \ldots, y_m) = 0]$ where p is a polynomial with integer coefficients and the variables range over natural numbers; later the work of Davis, Putnam, Robinson and Matiyasevich showed that one can replace the statement with something of the form: $\forall x_1 \ldots x_n \, [p(x_1, \ldots, x_n) \neq 0]$, For discussion, see Feferman (2006), p. 6.

of the truth of the statement about the solutions to a polynomial is itself simply a number-theoretical problem, it follows that each particular system, if it *is* consistent, cannot solve some mathematical problem; and that *if* human cognition is equivalent to some *particular* system then there is some problem of this form (equivalent to the statement of its own consistency) that *it* cannot solve either. This is then an "absolutely undecidable" problem. If, however, there is no formal system to which human cognition is equivalent, then for *any* specified machine the mind can prove a statement which that machine cannot, and accordingly "the human mind … infinitely surpasses the powers of any finite machine."[11]

Gödel's argument in the 1951 article also turns centrally on Turing's demonstration in 1937 of the unsolvability of the "decision problem" first suggested by Hilbert.[12] The question that Hilbert posed was whether it was possible to find finite algorithmic means to decide each well-defined mathematical "yes or no" question in finite time. In particular, Turing showed the unsolvability by finite algorithmic means of a particular "decision" problem, the so-called "halting" problem. The halting problem is the problem of finding a general procedure for determining whether any given algorithm (or Turing machine) will eventually halt, given any particular input, or will run on forever. As Turing demonstrated, there can be no such algorithm, since, supposing for *reductio* that one exists, it would then be possible to specify a Turing machine which halts if and only if it does not. The result implies not only (as Turing points out) the existence of real numbers whose decimal expansion is *uncomputable* (in the sense that there is no finitely statable procedure for determining the digits of the expansion) but also that first-order logic and stronger formal theories are *undecidable* in the sense that there is no finite *decision procedure* capable of determining, of any given formula, whether it is a theorem of the system or not.[13] As I shall suggest in the next several chapters, this systematic undecidability of formal, axiomatic systems has consequences for the "ontological" theory of the finite, the infinite, and their relationship to thought and practice. In particular, if the essential undecidability that Turing demonstrates henceforth marks a formally demonstrable limit to the effectiveness of formal procedures, this transforms in a basic way the question of the *accessibility* of the infinite to "finite" thought. If there is (as Turing's result demonstrates) no procedural means to decide the following of a given formula from a given formal system, and if (as Gödel's result demonstrates), for any such system (of enough strength to express arithmetic) there will be sentences that cannot be proven or refuted by any systematic means, then it is no longer possible *in general* to consider the truth of sentences to be decidable by any finite, procedural means.

[11] I follow here the trenchant and careful exegesis of Gödel's conclusion in the Gibbs lecture given by Feferman (2006), pp. 1-7. As Feferman notes (p. 7), there are a few auxiliary premises that are needed to assure the validity of Gödel's argument for the "disjunctive conclusion": first, that the human mind, in demonstrating truths, "only makes use of evidently true axioms and evidently truth-preserving rules of inference"; second, that these axioms include those of Peano Arithmetic; and third that a "finite machine", in the relevant sense, proves only theorems that are also provable by the human mind (or in other words that the power of a formal system is in any case no *greater* than that of the human mind).

[12] Essentially the same result was published by Church (working with a different formalism) in 1936.

[13] Undecidability in this sense – that of the undecidability of systems -- should be distinguished from the necessary existence, demonstrated by Gödel himself, of *sentences* in any system strong enough to capture arithmetic which are "undecidable" in the sense that the system cannot prove either the sentence or its negation.

Thought meta-formally in a broader philosophical context and relevantly to the problematic relationship of thought to being in itself, this suggests, as I shall argue in more detail, the essential limitation of the faculties, capabilities or capacities of a *representing subject* with respect to the inexhaustibility of the infinite-Real.[14] In particular, it suggests a decisive limit to the conception of the subject that begins in its modern form with Descartes and continues in Leibniz, Kant, Hegel, and German Idealism. Such a subject attains specific access to the infinite only by means of methodically specified rules or procedures applicable to the finitude of a kind of being constitutively limited in space and time, while at the same time the infinite itself is thought as the unformalizable excess of a divine-Absolute, which is thus inaccessible as such to human cognition. This twofold conception, which receives expression in Kant as the dualism of the finitely constituted subject of faculties which stands under the necessity of schematizing the deliverances of empirical affection under the categories, and the divine intellect capable an immediately creative intuition, is itself overcome in a twofold way by the complex of results that runs from Cantor to Turing. They do so by demonstrating, on the one hand, the actual accessibility of the mathematically infinite to formal thought and, on the other, the inherent limitation of finite procedures in attaining to it.

Besides thereby pointing to the formal and historical limits of the modern philosophy of subjectivity, Turing's result and the related ones to which Gödel appeals also, as I shall argue, have important implications for the ontological character and totalizing scope of what is called "information technology" today. In particular, if Turing thus demonstrates the inherent limits of the effectivity of formal procedures at the very moment at which he constructs the first formal definition of the structural architecture of a general computing machine, his result can be read as pointing to an ultimately inherent *ineffectivity* that thus accompanies the formalization of procedures and the imposition of "abstract" rule-based forms of reasoning and practice as their generally obscured but nevertheless structurally necessary underside. It is in terms of this specific structure of ineffectivity, as I shall argue in more detail, that the possibility of anything like a "reversal" or "overcoming" of the "metaphysical" essence of contemporary technology and its claim to global dominance can today be thought.

II

I term an *orientation of thought* a schema of the relationship between thought and being, identified by the form in which it modulates this relationship according to the "metaformal" ideas of consistency, completeness (or totality), and the reflexivity involved in thinking the totality from a position within it.[15]

[14] The issue can, again, be connected to that of the status of the most famous unsolved (and, as we now know, unsolvable) problem of set theory, Cantor's problem of the size of the continuum. From the work of Gödel himself in 1939 and Cohen in 1962-63, we now know that the continuum hypothesis (CH), which holds that the size of the continuum is the same as that of the first non-countable ordinal, cannot either be demonstrated or refuted on the basis of the standard ZF axioms of set theory. It is thus plausible that the question of the truth of the generalized continuum hypothesis is *one* of the many problems that should be seen as undecidable in an "absolute" sense.

[15] I here develop the conception of orientations of thought suggested by Alain Badiou in his 1998 *Briefings on Existence:*

> I call an "orientation in thought" that which regulates the assertions of existence in this thought. An orientation in thought is either what formally authorizes the inscription of an existential quantifier at the

Since each one involves that thought is thus related in some formally specifiable way to the infinity of thinkable beings, each one can be specified in terms of the specific figure of the infinite that it propounds. This allows us to specify the four orientations of thought according to these figures of the infinite, as well as the way they are produced by the specific relationship of the metaformal ideas in each case (figure). First, the *onto-theological orientation*, marked in traditional thought since Aristotle, supposes the infinite as a transcendent Absolute, complete and consistent in itself but inaccessible as such to finite, human cognition. Second, the *constructivist* orientation, original with Kant, propounds the infinite as always only *potential* in order to perform the delimitation of the totality of what can be known or thought according to regulative protocols or coherent criteria of finitude. But beyond these initial two, there are the two options opened by the Cantorian discovery of the transfinite as irreducibly plural and as permitting the inscription of the whole or totality only on pain of unavoidable paradox. These are the "generic" and the "paradoxico-critical" orientations, and they directly correspond, as well, to the two options left open by Gödel's own disjunctive conclusion in the 1951 lecture. On the first of Gödel's options (the one on which the human mind can recognize the truth of an infinite number of propositions beyond the powers of formal systems to demonstrate), this power witnesses an essential *incompleteness* of any finitely determined cognition and a correlative capacity on the part of human thought, rigorously following out the consequences of the mandate of consistency, to traverse the infinite consequences of truths essentially beyond the reach of any such finite determination. On the second of the options (that of problems forever unsolvable by formal *or* 'human' means), the essential *undecidability* of any such system witnesses, rather, the necessary *indemonstrability* of the *consistency* of any procedural means available to the human subject in its pursuit of truth, and thereby to the necessary existence of mathematical problems that are absolutely unsolvable by any specifiable epistemic powers of this subject, no matter how great.

> head of a formula, which lays out the properties a region of Being is assumed to have. Or it is what ontologically sets up the universe of the pure presentation of the thinkable. (Badiou 1998b, p. 53).

For a more detailed exposition of the four orientations of thought which I distinguish (adding one, the paradoxico-critical, to the three suggested by Badiou in 1998), see Livingston (2012), pp. 51-60.

	Critical Orientations	***"Dogmatic" Orientations***
Post-Cantorian Orientations	**Paradoxico-Critical**: Completeness, inconsistency	**Generic:** Consistency, incompleteness
Pre-Cantorian Orientations	**Constructivist**	**Onto-Theological**

The two "post-Cantorian" orientations are to be distinguished from the two "pre-Cantorian" on the basis of the specific relationship each envisions between thought and being in connection with the idea of the *totality* of *what is* (in Heidegger's jargon, "das Seiendes im Ganzen") as such. Both the onto-theological and constructivist orientations can be distinguished from the "post-Cantorian" ones in that they presuppose, though on different grounds, that this totality exists unproblematically in that it is possible for it to be both complete and consistent. In particular, whereas the onto-theological orientation assumes an infinite or transcendent consistent totality that is in itself never completely accessible to "finite" human cognition, the constructivist orientation constitutively involves the assumption of a knowable and consistent totality (for instance of what can be experienced or what can be referred to by means of a particular language) that is regulated, limited, or constructed by subjective procedures, activities, or forms. Because both of the "pre-Cantorian" orientations develop their respective conceptions of the thinkability of being in terms of the thought of a consistent and complete totality, they both amount to primarily *ontic* orientations toward what they figure as the totality of present-at-hand (*Vorhanden*) beings. Whereas onto-theology, understanding this totality as a complete and consistent whole quite independent of the capacities or activities of subjects or thinkers, captures most forms of realism that have been articulated in the metaphysical tradition, constructivism, in lodging the combination of consistency and completeness in the constitutive activities or abilities of a subject of experience, dialectical self-recognition, or linguistic institution, encompasses most forms of subjective, transcendental, speculative and linguistic idealism (or anti-realism).

If, however, the set-theoretical and semantic paradoxes already indicate the untenability of the conjunction of consistency and completeness that both "pre-Cantorian" orientations assume, then the orientation of thought toward being can subsequently only be thought in one of the two "post-Cantorian" ways: either in terms of the generic orientation, which preserves consistency while sacrificing completeness (thus maintaining the consistency of rules of inference while sacrificing the existence of a total world to which they apply), or the paradoxico-critical orientation, which develops the thought of the *constitutively inconsistent* whole. As we shall see, the impossibility of the joint ("pre-Cantorian") assumption of consistency and completeness receives further confirmation, and is put on a farther-reaching basis, in relation to the constitutive ideas of practice, method, and procedure, by means of Gödel's and Turing's undecidability results. These results, as we shall see, jointly bear witness to an essential undecidability at the limit of all possible procedures, methods or activities of human (or any) rationality, insofar as these procedures, methods and activities are determined by rules in any sense. The formally indicated limit of the powers of the thinking subject then becomes visible as the closure of the metaphysical assumption of complete and consistent totality, and thereby of the whole historical epoch that Heidegger understands as that of the "metaphysics of presence".

Because both of the "pre-Cantorian" orientations develop their understanding of the relationship between thinking and being on the basis of the presupposition of the completeness and consistency of the *ontic* totality, they are equally (and in parallel fashion) overcome by the thought of the ontological difference. As Heidegger recurrently emphasizes in his discussions of the metaphysical tradition, the various configurations and approaches of metaphysics, beginning with Parmenides, always have in view the ontic *totality* of present-at-hand beings and always think this totality on the basis of the assumption of its joint *consistency* and *completeness*. This assumption of consistency and completeness remains just as much in force in what I have called the constructivist orientation, where the complete and consistent totality of beings is thought, on the one hand, as the totality of objects conditioned by the constructive or limitative activities of a subject and on the other as the thinkable but unknowable totality of things in themselves. What Heidegger calls the history of "onto-theology" (in the broader sense in which it includes as well the constructivist orientation) is identical with the history of this assumption of conjoint completeness and consistency in the totality of beings, and so the ontological displacement that it undergoes through Heidegger's thinking of the ontological difference and its radicalized consequences also provides ontological terms for the transition from the "pre-Cantorian" to "post-Cantorian" orientations.

The metalogical results thus verify, when situated in terms of the four orientations of thought, a structural convergence of "metalogical" difference with the ultimate implications of what Heidegger calls "ontological" difference. In particular, the difference that insists in the generation of the two post-Cantorian orientations and their distinction from one another – that between consistency with incompleteness and totality with inconsistency – can also be seen as a figure or articulation of the ontological difference, once the "relationship" between being and beings is itself understood in terms of the metalogical results. Here, it is less important to determine which of the two post-Cantorian orientations Heidegger "himself" occupies than to see how the ontological difference can itself be articulated in both ways and how the two articulations complement one another. First, there is the generic position, which maintains consistency by situating thought with relation to an always-external matter than insists in it at the point of the truth it properly reveals. Second, there is the paradoxico-critical one, which sees the closure of the totality of beings only in the light of a fundamental structure of paradox which always again arises at the boundaries of any complete determination of it. Understood ontologically and in the context of the "history of being", the two figures of thought are complementary to one another: the first points to that which, in determinate total configurations of beings as a whole, points beyond their determining (ontic) figure to their ultimate dispensation in "being itself", while the second points to the structurally necessary paradox which, in every such configuration, undermines the assumed consistency of its basis and organizing structure. Together, they articulate jointly and formally the meta-structure of the configurations in which being "grants" itself in the various epochal configurations, the more general formal structure of the epochal determination of the successive figures of the being of beings itself. For the same reason, they articulate as well the formal structure of the contemporary historical "closure" of the metaphysics of presence itself in relation to what surrounds it as its more general ontological condition.

It is also true that Heidegger does not generally distinguish, in his own accounts of the history of metaphysics up to the present, the two "pre-Cantorian" orientations which characteristically neglect both ontological and metalogical difference: the "onto-theological" position (in our sense) which situates the consistent totality in a divine intellect or cosmological unity beyond the powers of finite or human thought and the constructivist one that identifies positive existence as what is thinkable by the finite subject. This unclarity in Heidegger's own retrospective discussion is, doubtless, one root of the contemporary interpretive tendency which, while grasping to some extent the terms and implications of Heidegger's critique of onto-theology, nevertheless continues to assimilate to him some form of the thesis that assimilates the "accessibility" of being as such to the powers of a thinking (individual or collective) subject. Heidegger then can only appear as a kind of post-Kantian anti-realist, and of course it is not difficult to locate in his writing, particularly in *Being and Time*, the conception of a structural "transcendence" which, as in Kant, is then thought to underlie the constructive relationship of something like a "human" subject – albeit now the living and factical subject of "emdodiment" and practices rather than the "intellectual" or "worldless" subject of Descartes, Kant, and (it is supposed) Husserl – to entities or their being. The conception has the additional merit of conforming well with a prevalent (and essentially constructivist) conceit of contemporary belief, according to which, if there is no "ultimate" theological referent to hold together the totality of the world as an intellectually thinkable unity, such access to being as it is possible "for us" to have must instead be facilitated, in irreducibly pluralistic fashion, by the variety of bodies, languages, and situated cultures.

As we shall see in this chapter and the next one, however, an explicit identification of the metalogical issues at stake in the four orientations, and in particular to the inherent and essential meta-formal realism essentially presupposed in *both* of the post-Cantorian ones, points to a very different conception of the ontological problematic that is already suggested by Heidegger in *Being and Time*, albeit only developed fully in the course of his radical critical encounter with Kant in *Kant and the Problem of*

Metaphysics and his subsequent development of the "grounding" question of the truth of being in contrast to the "guiding question" of beings. On this conception of the ontological problematic, it deconstructs the idea of a constitutively essential basis for ontology in "human" thought, language, culture, embodiment and practices just as thoroughly, and on substantially the same basis, as it does the "theological" intuition of the transcendent Absolute. As such, this conception, as we shall see, substantially underwrites the possibility of a *realist* conception of the ontological difference and of the ontological structure of time.[16] In particular, this underlies, as I shall argue, the possibility of an *ontological* realism – that is, a realism with respect to "being" itself – as well as a correlative *temporal* realism which suffices to displace any figure of time as originating in subjective consciousness, intersubjective practice, or subjectivity in general.

Much of the discussion in the extent philosophical literature over the broader implications of Gödel's theorems has been directed toward the question of the truth or falsity of *mechanism*, or of whether the mathematical thought of an individual subject, or perhaps of the whole community of mathematicians, can "in fact" be captured by some formal system. It is not clear, despite this discussion, that the problem can be well posed, and even if it can, it appears possible on the basis of Gödel's results only to draw a "disjunctive" conclusion of the same form as the one he draws explicitly in the 1951 lecture: either human minds cannot be captured by formal systems and so have (somewhat mysterious) powers to perceive or demonstrate truths beyond the scope of finite, algorithmic procedures *or* "minds are machines" after all and there are problems that are *absolutely* unsolvable by either. Nevertheless, beyond the issue of mechanism, it is also possible to see the upshot of Gödel's "disjunctive conclusion" in the lecture as bearing relevance to broader and different philosophical issues.[17] In particular, it points to a distinctive and non-standard, but comprehensive position of *realism*, what I shall call *meta-formal* realism.[18] For this realism, the decisive issue is not, primarily, that of the reality of "mathematical objects" or the possibility of understanding them as determinate independently of the routes of access to them (epistemic or otherwise) involved in the exercise of our human capacities. It is, rather, that both terms of Gödel's disjunction capture, in different ways, the structural point of contact *between* these capacities and what must, on *either* horn of the distinction, be understood as an infinite *thinkable* structure determined quite independently of anything that is, in itself, finite. Thus, each term of Gödel's disjunction reflects the necessity, given Gödel's theorems, that

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[16] Gödel himself, particularly in his later years, was, as is well known, a dedicated anti-mechanist, and sometimes referred to his incompleteness theorems as providing evidence against mechanism; more recently, philosophers such as Lucas and Penrose have followed Gödel in arguing for this conclusion. However, see Shapiro (1998) for an overview of the issues about mechanism and the conclusion that "there is no plausible mechanist thesis on offer that is sufficiently precise to be undermined by the incompleteness theorems." (Shapiro (1998), p. 275).

[17] I thus follow Feferman (2006), p. 11 in considering that, even if there are problems with applying Gödel's reasoning directly to the question of mechanism, "…at an informal, non-mathematical, more every-day level, there is nevertheless something to the ideas involved [in his argument for the "disjunctive conclusion"] and something to the argument that we can and should take seriously."

[18] In *The Politics of Logic* (Livigston 2012, p. 291), I called this position simply "formal realism". I add the prefix 'meta-', here, to reflect that what is concerned is not primarily an attitude (e.g. a Platonist one) about the "reality" or "actual existence" of forms, but rather the implications of the *transit* of forms in relation to what is thinkable of the real, the transit that can, in view of Cantor's framework, be carried out beyond the finite.

any specification of our relevant capacities involve their relation to a structural infinity about which we must be realist, i.e. which it is not possible to see as a mere production or creation of these capacities.

On the first alternative, this is obvious. If human mathematical thought can know the truth of statements about numbers which are beyond the capacity of *any* formal system to prove, then the epistemic objects of this knowledge are "realities" (i.e. truths) that also exceed any finitely determinable capacity of knowledge. It does not appear possible to take these truths as "creations" of the mind unless the mind is not only credited with *infinite* creative *capacities*, but understood as having actually already *created* all of a vastly infinite and in principle unlimitable domain. But on the second alternative, it is equally so. If there are well-specified mathematical problems that are not solvable by any means whatsoever, neither by any specifiable formal system nor by human cognition itself, then *these problems* must be thought of as realities determined quite independently of our capacities to know them (or, indeed, to solve them).[19] On this alternative, we must thus acknowledge the existence of a reality of forever irremediable problems whose very issue is the inherent undecidability that results from the impossibility of founding thought by means of an internal assurance of its consistency. In this way the implications of the mathematical availability of the infinite, on either horn of the disjunction, decompose the exhaustiveness of the situation underlying the question of realism and idealism in its usual sense: that is, the question of the relationship of a presumptively finite thought to its presumptively finite object.

The actual underlying reason for the realism which appears forced upon us on either alternative is the phenomenon Gödel describes as that of the *inexhaustibility* of mathematics, which results, as we have seen, from the possibility of considering, given any well-defined ordinal process, its infinite limit (or totality). On the first alternative, this inexhaustibility yields a structurally necessary *incompleteness* whereby each finite system by itself points toward a truth that it cannot prove but which is nonetheless, by this very token, accessible to human thought. On the second, it yields an equally necessary *undecidability* which leaves well-specified mathematical problems unsolveable by any means (finitely specified or not) by any means whatsoever.[20] The form of the relevant realism is, in each case, somewhat different: the orientation underlying the first disjunct corresponds, as I argued in *The Politics of Logic*, to a realism of *truth beyond linguistic sense*, a position that affirms the infinite existence of truths and the infinite genericity of our dynamic insight into them beyond any finitely specifiable language or its powers, while the realism of the second consists is a realism of *sense beyond linguistic*

[19] Gödel says this about the second term of the disjunction: "… the second alternative, where there exist absolutely undecidable mathematical propositions, seems to disprove the view, that mathematics (in any sense) is only our own creation…So this alternative seems to imply that mathematical objects and facts or at least *something* in them exist objectively and independently of our mental acts and decisions, i.e. to say some form or other of Platonism or "Realism" as to the mathematical objects." (Gödel 1951, pp. 135-36).

[20] Feferman (1962) has shown that there is a kind of "completeness" of arithmetic truths that is obtainable by the transfinitely repeated application of so-called "reflection principles", each of which amounts to adopting as a new axiom for a new system certain assumptions about the consistency of an earlier system (or the truth of its results). By means of an appropriate transfinite procedure through these principles, it is indeed possible, as Feferman shows, to obtain the totality of arithmetic truths. However, this procedure is itself not specifiable in a recursively enumerable way, and so does not provide anything like a general effective procedure for determining arithmetic truth. See Shapiro (1998) and Berto (2009) for discussion.

truth, affirming the existence of linguistically well-defined *problems* whose truth-value remains undecidable under the force of any powers of insight whatsoever. But in either case, reflective thought about human capacities must reckon with the consequences of their structurally necessary contact with an infinite and inexhaustible reality essentially lying beyond the finitist determination of the capacities of the human subject or the finitely specifiable powers of its thought. In this way, the consequences of Gödel's theorem, however we interpret them, engender a structurally necessary realism about the objects of these powers that is the strict consequence of the entry of the infinite into mathematical thought.

As Gödel immediately goes on to point out, the only position from which it appears possible (while accepting Gödel's assumptions about mathematical reasoning and the incompleteness theorems themselves) to resist the "disjunctive conclusion" is a *strictly finitist* one according to which "only particular propositions of the type 2+2=4 belong to mathematics proper…"[21] and no *general* judgments applying to an infinite number of cases are ever possible. This kind of position would indeed avoid the disjunctive conclusion, since there is no way to apply the incompleteness theorems themselves consistently with it. However, as Gödel points out, the strict finitist view is very implausible as a view of mathematical reasoning, since it ignores that "it is by exactly the same kind of evidence that we judge that 2+2=4 and that a+b=b+a for any two integers a,b"; and it would moreover appear to disallow the use of even such simple "concepts" as "+" (which "applies" to all pairs of integers). Outside these very severely limited finitistic point of view, on the other hand, it appears inevitable that the disjunctive conclusion will apply, and thus we will be forced to acknowledge the validity of one or both of its disjuncts.

The attitude I am calling "metaformal realism" might certainly be developed as a position within the philosophy of mathematics itself. Developed in this way, it would bear a resemblance to a "methodological" realism about mathematics, for example of the kind suggested by Maddy (2005), that characteristically looks to mathematical practice itself as the source for its "ontological" claims and assumptions. As Maddy suggests, this kind of realism has the advantage that it does not entertain, or attempt to solve, "metaphysical" problems about the "existence" of mathematical objects, except insofar as these problems are formulable, in a motivated way, within mathematical practice itself (here, including the kind of "metamathematics" or "metalogic" that Gödel uses to produce his incompleteness theorems). Because of the way that it thus treats the problems, as they develop in practice, as primary, this kind of attitude should also be distinguished from "Platonism" as it is traditionally construed in the philosophy of mathematics. In particular, as Badiou (1998) has argued from a similar perspective, there is no need to invoke, even in service of a realist attitude that here takes the thought of the infinite and the consequences of mathematical practice seriously, the "Platonistic" claim of the "real existence" of mathematical objects. What is at issue is rather the articulation of problems relating to the consistency,

[21] Gödel (1951), p. 135.

decidability and *truth* of statements ranging over infinite domains of quantification, including decisively those which refer to empty domains or actually threaten to produce inconsistencies.[22]

At the same time, though, the broad attitude of metaformal realism appears to have many implications beyond strictly mathematical thought or practice itself. For the consequences of formalism and formalization themselves in their contemporary practical and theoretical development are by no means limited to mathematics, but extend to a broad range of phenomena and many aspects of contemporary social and political life. As a leading example of this (though there are certainly others) one might consider the pervasiveness of informational and computational technologies and the forms of abstract social organization they make possible, themselves grounded in the technology of the computing machine which was directly made possible by the development of the implications of the concept of a formal system in thinkers such as Hilbert, von Neumann and Turing. The specific relevance of mathematics and metamathematics, in this connection, does not lie in the identification of a particular realm or region of entities, but rather in the way that mathematics, *as* the "science of the infinite", possesses the ability to capture and schematize the constitutively "infinite" dimension of form itself. This infinite dimension of forms is a constitutive part of the thinking of form, even when it is dissimulated or foreclosed, ever since Plato, and is inherently involved, as well, in every contemporary project of the analysis of logical form or the discernment of the formal determinants of contemporary life and practices. Most broadly, it provides a formal basis for interrogating the constitutive idea of the rational human subject or agent as definable in terms of its finitely specified capacities. With this basis, it is possible to transpose the (pre-Cantorian) Kantian figure of opposition between the finitude of sensory affection and the absolute-infinite divine intellect capable of intellectual intuition. In its place, the metafomral reflection offers to reinvent the possiblities of critique on the ontological real ground of the objective undecidability of problems that are problems for (finite or infinite) thought *in itself*, given to it at the point of its very contact with the real of being as such.

What, then, are some of the concrete effects of this transposition for contemporary reflective and critical thought? As I argued in *The Politics of Logic,* most generally, the necessity, in a post-Cantorian context, of the forced choice between inconsistent completeness and incomplete consistency indicates, as is confirmed by Gödel's development of the philosophical consequences of his own results, that it is impossible *by* finite, procedural means to confirm rigorously the *consistency* of the finitely specifiable

[22] More specifically, Badiou argues in a related context (1998b) that the "Platonist" attitude of object-invoking realism is in fact quite alien to Plato's own concerns; in particular, it relies upon a "distinction between internal and external, knowing subject and known 'object'" which is, as Badiou says, "utterly foreign" to Plato's own thought about thought and forms. Plato's fundamental concern is not, as Badiou argues, at all with the question of the 'independent existence' of mathematical objects, but rather with the 'Idea' as the name for something that is, for Plato, "always already there and would remain unthinkable were one not able to 'activate' it in thought." (Badiou 1998b, p. 49). Similarly, this is, as Badiou emphasizes, not an attitude of accepting or *believing in* the existence of sets or classes corresponding to well-defined monadic predicates, but rather one of maintaining, quite to the contrary, that what correlates to a well-defined concept may well be "empty or inconsistent"; it is thus a *metalogical* inquiry into the structure of forms for which, as Badiou emphasizes, "the undecidable constitutes a crucial category" and in fact becomes the central "reason behind the aporetic style of the [Platonic] dialogues," wherein thought constantly proceeds through forms to their own inherent points of dissolution or impasse.

procedures of our social-political, practical, and technological worlds. This suggests, as I argued at more length in the book, that it is impossible by finite means to ensure the *effectivity* of our practices, or procedurally to found whatever faith we may maintain in their ongoing extensibility and capability of continuation. This faith, if it is to be founded at all, must be founded in an essentially infinite capacity of insight and fidelity, bordering on the mystical, to a Real matter of consistency with respect to our own practices that can itself never be guaranteed by any replicable or mechanical procedure; *or* it must be ceaselessly decomposed and deconstructed at the point of the inherent realism of the problematic and undecidable that is necessarily introduced if this faith cannot be assured at all.

III

In contemporary philosophical discourse, no project has done more to illuminate the issue of realism and its underlying formal determinants than Michael Dummett's. Familiarly, in a series of articles and books beginning in 1963 with the article "Realism," Dummett has suggested that the dispute between realism and anti-realism with respect to a particular class of statements may be put as a dispute about whether or not to accept the principle of *bivalence* (i.e., the principle that each statement is determinately true or false) for statements in the class concerned.[23] Though this issue yields differing consequences in each domain considered, the acceptance of bivalence generally means the acceptance of the view that all statements in the relevant class have truth values determined in a way in principle independent of the means and methods used to verify them (or to recognize that their truth-conditions actually obtain when they, in fact, do so); the anti-realist, by contrast, generally rejects this view with respect to the relevant class. Dummett did not envisage that this comprehensive framework would or should support a single, *global* position of metaphysical "realism" or "anti-realism" with respect to all domains or the totality of the world; rather, his aim was to illuminate the different kinds of issues emerging from the traditional disputes of "realism" and "idealism" in differing domains by submitting them to a common, formal framework.[24] From the current perspective, however, it is just this aspect of formal illumination which is the most salutary feature of Dummett's approach. For by formally determining the issue of realism with respect to a given domain as one turning on the acceptance or nonacceptance of the (meta-)formal principle of bivalence with respect to *statements*, Dummett points toward a way of conceiving the issue that is, in principle, quite independent of any *ontological* conception of the "reality" or "ideality" of *objects* of the relevant sort. In particular, it is in this way that Dummett avoids the necessity to construe realism and anti-realism in *any* domain as involving simply differing attitudes toward the ontological status of its objects (for instance that they are "mind-independent" or that, by contrast, they are "constituted by the mind"). What this witnesses, *along with* what I have called meta-formal realism, is the possibility of a purely formal and reflective determination of the issue of realism that connects its stakes directly to those of the truth of claims, thereby instantly

[23] Dummett (1963); for some later reflections on the development of the framework and issues related to it, see Dummett (1978).
[24] Dummett (1978), pp. xxx-xxxii.

short-circuiting the laborious and endlessly renewable dialectic of the "actual relationship" of mind to world.

Dummett's framework is sometimes glossed in terms that suggest that, for him, the adoption of realism or anti-realism in any particular case turns primarily on our judgment about the (primarily epistemological) issue of whether a certain type of entities can be considered to be real in themselves, independently of our access to them or ability to possess evidence for their existence. But that this kind of formulation is, at best, highly misleading, both with respect to Dummett's own motivations and the actual merits of the framework he recommends, can be seen from the introductory formulation of the issue of realism and anti-realism in the original article "Realism" itself:

> For these reasons, I shall take as my preferred characterisation of a dispute between realists and anti-realists one which represents it as relating, not to a class of entities or a class of terms, but to a class of *statements*, which may be, e.g., statements about the physical world, statements about mental events, processes or states, mathematical statements, statements in the past tense, statements in the future tense, etc…[T]he realist holds that the meanings of statements of the disputed class are not directly tied to the kind of evidence for them that we can have, but consist in the manner of their determination as true or false by states of affairs whose existence is not dependent on our possession of evidence for them. The anti-realist insists, on the contrary, that the meanings of these statements are tied directly to what we count as evidence for them, in such a way that a statement of the disputed class, if true at all, can be true only in virtue of something of which we could know and which we should count as evidence for its truth. The dispute thus concerns the notion of truth appropriate for statements of the disputed class; and this means that it is a dispute concerning the kind of *meaning* which these statements have.[25]

There are two points here that bear important implications for the issue of how best to characterize realism and anti-realism. The first is that, on Dummett's formulation, it is an issue, not of the reference of terms or the existence of objects, but of the way in which the truth-values of statements are determined. The second, following from the first, is that the question of realism within a given domain is not directly an epistemological question about our knowledge of (or 'access to') entities, but rather a semantic question about the basis of the *meaning* of statements. As Dummett points out, both points are helpful in characterizing the real underlying issue and separating it from other issues that have become confused with it in the history of discussion of realist and idealist (or nominalist and universalist, etc.) positions. For example, in the traditional debate between phenomenalists and realists about material objects, which has sometimes been put as a debate about their "existence", Dummett argues that his framework allows the actual question of realism to be separated from what is in fact a conceptually different one, the question of *reductionism* (i.e. of whether 'material objects' can in fact be reduced to something like sense-data). Somewhat similarly, with respect to mathematics, concentrating on the question of the reference of terms tends, Dummett suggests, to "deflect the dispute from what it is really concerned with"; in particular, "the issue concerning platonism relates, not to the existence of

[25] Dummett (1963), p. 146.

mathematical objects, but to the objectivity of mathematical statements."[26] Here again, a framework primarily directed toward the question of the meaning of statements is more useful than one concerned primarily with questions of the existence of objects. This is, at least in part, because in mathematics (as opposed to some other cases) it is generally implausible to suppose we can have "access" to the relevant "objects" independently of a recognized procedure (i.e. a calculation or a proof) for establishing the truth of statements about them; and on the other, that such a procedure is also generally taken to be *sufficient* for whatever access to mathematical objectivity we can enjoy.

Although this kind of consideration finds application quite generally, it is certainly no accident that the historical dispute which forms the basic model for Dummett's formal framework itself is the dispute between formalists and intuitionists about the foundations of mathematics in the 1920s and early 1930s. Partisans of the two positions reached deeply opposed conclusions about the nature of reasoning about the infinite, but for both positions the idea of a *finite (i.e., finitely specifiable) procedure* or process of demonstration plays a central role. In particular, whereas the formalist position allows the axioms and rules of a formal system to extended classically, by means of such a procedure, to arbitrarily extended reasoning about the infinite *provided* that the system can be shown to be consistent, intutionism generally restricts the positive results of mathematics about the infinite to what can be shown by means of a finite, constructivist procedure of proof.

In the 1973 article "The Philosophical Basis of Intuitionistic Logic," Dummett considers the question of what rationale might reasonably serve as a basis for replacing classical logic with intuitionistic logic in mathematical reasoning (hence, in his framework, for replacing realism with anti-realism).[27] As Dummett emphasizes here, the decision between realism and anti-realism depends ultimately on our conception of how *sense* is "provided" for mathematical statements, and in particular whether we can conceive of these statements as having sense quite independently of our means of recognizing a verification of them. It is thus, ultimately, as Dummett sees it, general issues about the *capacities* or *practices* that we learn in learning a language and deploy in speaking one that determine, given his framework, equally general issues about whether realism or anti-realism is better justified in any given domain. As in the earlier article "Realism," Dummett here emphasizes that this primary issue is not an epistemic or ontological, but rather a semantic one. Thus, "Any justification for adopting one logic rather than another as the logic for mathematics must turn on questions of *meaning*"; and again, "it would be impossible to construe such a justification [i.e. for adopting classical or intuitionistic logic] which took meaning for granted, and represented the question as turning on knowledge or certainty."[28]

In fact, Dummett suggests, there are just two lines of argument that could plausibly be used to support the replacement. The first turns on the idea that "the meaning of a mathematical statement determines and is exclusively determined by its *use*"; beginning from this assumption, it is plausible to hold that any difference between two individuals in their understanding of mathematical symbolism would have to be manifest in observable differences of behavior or capacities. The second turns on considerations about

[27] Dummett (1973).
[28] Dummett (1973), p. 215.

learning, and in particular on the thought that what it is to learn mathematical reasoning is to learn how to *use* mathematical statements (i.e. when they are established, how to carry out procedures with respect to them, how to apply them in non-mathematical contexts, etc.).[29] On either assumption, it is then reasonable, Dummett suggests, to hold that *since* meaning is exhausted by use (in one way or the other) we cannot claim that a notion of truth, understood classically as imposing bivalence on all mathematical statements independently of the use we actually make of them, can any longer serve as the "central notion" for a characterization of the meanings of mathematical statements. In place of the classical notion of truth, Dummett suggests, we must substitute a notion grounded in the practices of which we have actually gained a mastery; in particular, we must replace the classical notion of truth with the claim that "a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us."[30] This, in turn, allows the recognition that certain classical arguments and proof-procedures are unjustified from this perspective, and should accordingly be replaced with intuitionistic ones.

Dummett thus presents the best route to the adoption of intuitionistic logic in mathematics as motivated by considerations very different from those that motivated arguments to the same conclusions for classical intuitionist thinkers such as Brouwer and Heyting. In particular, as Dummett points out, whereas intuitionism was motivated for those thinkers primarily by the requirement that mathematical objects be present or given in subjective, private experience, Dummett's arguments turn on what is in some ways the exactly opposite idea, namely that of the mastery of a socially learned and publically evident *intersubjective practice*. In fact, Dummett suggests against the views of the early intuitionists, there is *no* plausible route from the view that mathematical entities such as natural numbers are "creations of human thought" to the application of intuitionistic logic, unless we are prepared to adopt a very severely restricted (and implausible) view of mathematical practice (including rejecting unbounded quantification over all numbers, etc.)[31] For this reason, Dummett suggests as well that there is no good reason to think that any successful argument for anti-realism in mathematics can turn on considerations bearing simply on the supposed ontological peculiarities of the mathematical domain; both of the reasonable arguments that one might make turn, instead, on considerations about the link between meaning and use which have nothing special to do with mathematics and would seem to be applicable much more broadly, to any number of classes of sentences "about" widely differing kinds of things.

By posing the issue of realism vs. anti-realism, not only in the mathematical case but more generally, as turning on the question of the provision of sense, Dummett shows that the question of realism in a particular domain is most intimately related, not to the question of the ontological status of, or our epistemological access to, its objects, but rather to the question of the coherence and range of the procedures by means of which the *meanings* of statements about the domain are learned and manifested. But in the current context, this is none other than, again, the question of the way that the infinite becomes available on the basis of a finite procedure. For the intuitionist (and by analogy, the

[29] Dummett (1973), pp. 216-217.
[30] Dummett (1973), p. 225.
[31] Dummett (1973), p. 247.

anti-realist more generally), it is possible to establish the existence of an object only if it can be shown to result from its actual construction in a finite number of steps or from a finite, constructivist proof (i.e. one that does not involve reasoning over arbitrarily complex infinite totalities). By contrast, for the formalist (realist), all that is needed is to show that it is possible to refer to the object without contradiction within a specified formal system. And it is just here, with regard to the specific question of what is involved in the learning and pursuit of a finite procedure, that the possibility of meta-formal reflection of the sort that I have portrayed Gödel as engaging in proves to be decisive. For Gödel's own incompleteness theorems, of course, result directly from a rigorous meta-formal consideration of the range and capacities of formal systems (in Hilbert's sense and related ones). In particular, Gödel's first theorem shows that for any such system, there will be a number-theoretical sentence that is beyond its capacity to prove or refute, and the second theorem shows that no such system can prove its own consistency (assuming that it *is* consistent). In this way Gödel's results render the formalist conception of finite procedures unsuitable for anyone who wishes to assert the realist position that the statements of number theory have determinate truth-values, independently of our ways of verifying them; but on the other hand, in invoking under the heading of the "inexhaustibility" of mathematics an essential reference to a reality that marks the point of impasse of any given finite procedure, Gödel's argument shows the intuitionist strictures to be untenable as well.

Just as Gödel's theorems themselves thus overcome the debate between intuitionism and formalism, narrowly construed, by conceptually fixing and reflecting upon the contours of a central concept (that of a finite procedure) commonly appealed to by both, the meta-formal realism I have discussed as suggested by Gödel's argument provides a new basis for critically interrogating the central concept of a *rule of use*, as it figures in both "realist" and "anti-realist" conceptions of the structure of language. In particular, we may take considerations analogous to those which establish Gödel's second incompleteness theorem to demonstrate, the incapacity of a finitely specifiable system of such rules to establish its *own* consistency. It is then apparently possible to draw, with respect to our actual practices and institutions of linguistic use, a conclusion directly analogous to that drawn by Gödel with respect to mathematical reasoning specifically: namely that *either* the consistency of our regular practices can only be known, and assured, by a deliverance of an essentially irregular insight that essentially cannot be subsumed within them or determined by them insofar as they can be captured by rules; *or* it cannot be known at all and thus can *only* be treated as a perpetually deferred problem. On either assumption, the claim of consistency is shown to be, from the perspective of the regular provision of sense, the point of an impossible-Real that always escapes, drawing along with it any possibility of an internal systematic confirmation of the infinite noncontradictory extensibility of the rule to ever-new cases. It is in this way, as I have argued, that the phenomenon that Gödel calls the "inexhaustibility of mathematics" points toward a metaformally justified realism of the impossible-Real, correlative to what we may describe as our essential openness toward the infinite and based in metaformal reflection about the limits and transit of forms. In so doing, it unhinges any possible claim of the humanistically conceived "finite" subject finally to ground itself, or to secure by its own means the ultimate sense of its language and life.

What, then, of Dummett's own arguments for anti-realism in various domains? The general form of this argument is the one we have already seen with respect to mathematics. It turns most centrally, as we have seen, on the question of how sense is "provided" for the range of statements characteristic of the entities of a given domain. In particular, on the assumptions about the basis of sense that Dummett attributes to the late Wittgenstein (correctly or incorrectly), sense must be provided or established for any range of sentences by means of the establishment and learning of a (public, intersubjective) *practice*. On Dummett's various arguments, this makes it incoherent, in a variety of domains, to suppose that sense could have been provided or determined independently of procedures for verifying truth in those domains. According to Dummett, it is for this reason, for instance, that one must be anti-realist about descriptions of "private" experience, and these considerations at least suggest anti-realism about the past (though as Dummett admits, matters are more complex here owing to the internal complexity of the notion of the (current) verification of (past) events itself). The general argument goes through, as we have seen, on the assumptions that i) sense is provided by means of a practice which essentially *involves* laying down various well-defined procedures of verification and ii) this provision of sense is *not intelligible except by means* of the specification and establishment of these procedures. However, these assumptions are at least contestable in a context where (as I have argued with respect to the kind of metaformal reasoning Gödel applies) sense appears to be "provided" through finite instances of teaching and learning, but *in such a way* as to essentially *outstrip* any description of the outcome of these instances as the internalization of finite *procedures*. And truth here is demonstrated, not by any simple application of established verification procedures, but on a constitutive *reflection* on their scope and limits that is itself *irreducible* to any antecedently given procedure. More generally, since Dummett's argument turns on the thought that sense, if it is to be determinate, must first be "provided" by the human activity of instituting such procedures, it can be resisted where we have good independent reason to consider sense to be "given" in a way that essentially outstrips these procedures. Such an independent reason is provided, as I have argued, by the metaformal reflection that underlies Gödel's results and thereby demonstrates what he calls the "inexhaustibility" of mathematics, and more generally by the problematic accessibility of the infinite and transfinite to thought that is broadly witnessed in the results of Cantor, Gödel and Turing.

More narrowly, Dummett has at least sometimes suggested an argument from Gödelian incompleteness itself to anti-realism about mathematics. The argument is that, since as Gödel shows there are undecidable sentences for every consistent formal system capable of expressing arithmetic, there must be sentences, for any such system, that cannot be verified as true or false by means of its proof procedures. *Assuming* that truth an falsehood are intelligible *only in terms of intra-systematic proof*, there must then be, for any such system, sentences that are neither true nor false, and it may be thought to follow as a corollary that an intuitionist logic must therefore be adopted to treat them. This conclusion is, of course, very different from the disjunctive one Gödel himself draws from his incompleteness results, whose two alternatives must both be understood, as we have seen, as robustly realist. In fact, to argue *from* undecidability to intuitionist logic in the way that Dummett at least sometimes suggests is in a certain way to ignore the deeper underlying reasons *for* undecidability. For the argument involves *assuming* that proof can only be intelligible as an intra-systematic notion; whereas Gödel's first theorem itself depends on "verifying" the undecidability of the Gödel sentence for a particular system from an essentially *extra*-systematic perspective. And the possibility of this verification bears witness to that of a kind of "insight" into reality that is not simply the outcome of a proof procedure in any particular formal system. If we can assume that the Gödel sentence for a particular system, construed simply as an arithmetical sentence, has reference to the "actual" natural numbers, it is natural to put this "insight" as the insight into a truth about them that the system in question essentially cannot prove (if it is consistent). But in fact to appreciate the more general possibility of a genuine extra-systematic insight into the Real here, it is not necessary to conclude, as Gödel himself most often does, that the Gödel sentence for a particular system is an arithmetical truth in this sense. Even on the other horn of the disjunctive conclusion, where what is demonstrated is not a particular arithmetic truth but simply the undecidability of a sentence for a particular system, *this* undecidability is still demonstrated as a positive fact about this system (again on the assumption that it is consistent), and it is also possible to draw the more general conclusion that *every* formal system will evince some such sentence. What is gained with this insight, even if it is not a successive insight into the "truths" about natural numbers, conceived "Platonistically" as existences in and of themselves, is nevertheless a general meta-formal insight into the real limitations of all regular formal systems as such insofar as they are capable of touching on truth. On this horn of the disjunction, it thereby points, as I have argued, to an irreducible insistence of problems unsolvable by any procedural means, an insistence that must itself be considered an irreducible mark of their reality.

With respect to this problematic insistence of the Real at and beyond the limits of finite procedures, which shows up, as I have argued, only when the very idea of a finite procedure is subjected to critical and meta-formal reflection, along with the constitutive ideas of the finite and of capacities that underlie it, the question that Dummett characteristically asks about the relationship between the instituting or specifying of "use" and that of procedures of verification is thus not the most telling one. Rather, the real question is prior to this: it is the question of how *any* institution – or its communication in teaching or learning – can suffice to determine an infinite totality of truths about objects in a certain domain to begin with. But this is just the question underlying Wittgenstein's "rule-following" considerations in the *Investigations*, and Dummett's failure to discern the tension between Wittgenstein's critical inquiry into the very idea of rule-following and the "official" conception of "meaning as use" marks a real limitation of his reading of him.[32] As Wittgenstein argues, it is evidently no answer to this question to hold that the infinite number of truths about objects in a particular domain are determined "all at once" by means of the inscription of a symbolic formula or an experiential intuition of its "meaning." To say that language, or the use of a word, is a "practice" or "institution" is thus not to say that it is determined or determinable, once and for all, by means of a finitely specifiable regular procedure specified or specifiable in advance; but rather that its form is shown in *what we do* from case to case of new applications.[33] Specifiable "procedures" for verification are given along with this general form as it is lived, and as emerge from it by means of explicit formal reflection. But this does not mean that the understanding of the truth or falsity of claims that is shown in these instances and given in this form must be constrained by them in such a way that it is not possible to maintain their "realist" reference to

[32] This failure is doubtless responsible for Dummett's attribution to Wittgenstein, in a famous and critical review (Dummett 1959) of the latter's *Remarks on the Philosophy of Mathematics* of a "full-blooded" conventionalism according to which the result of a new calculation must be spontaneously decided in each case.
[33] Wittgenstein (1953), §§198-202.

things as they are in themselves and to the correlative determinacy of truth and falsehood.[34] Moreover, as the results of Gödel and Turing show, there in fact *must* be, given any formal procedure of verification of a certain type, "realist" truths that cannot be established thereby, whether of the actual truth-value of propositions, e.g. of the Gödel type, or about the irreducible insistence of *problems* that, though completely determinate in themselves, essentially evade *any* procedural/regular solution.

Viewed this way, the realism that is recommended on either horn of Gödel's disjunctive conclusion in fact has implications far beyond the domain of "mathematical" truth itself. In particular, a similarly motivated meta-formal realism is recommended wherever it is possible (or suggested on independent grounds) to be realist about *sense* itself. We obtain this realism as soon as we recognize that the *constitution* of sense in any particular domain is not simply the result of its *construction*, whether by means of the capacities or activities of a finite "human" subject or indeed by the institution of finitely specifiable regular procedures, rules, or norms of "intersubjective" practice. As we have seen, Dummett's framework has the salutary benefit of resituating the question of realism as a question of the determinacy of sense rather than as the old question of the constitution of objects in relation to our ways of knowing about them. But in applying it and especially in arguing for anti-realism, he tends to assume that sense *must* be "provided" by means of socially instituted practices, if it is to be provided at all.

By contrast with this, metaformal realism is a realism about the "provision" or constitution of sense that separates it from any constructivism, whether of a subjectivist, social-pragmatist, intuitionist or any finitist kind. From the current perspective, the problem of this constitution is not distinct from the problem of the accessibility of the infinite to thought itself. For (linguistic) sense is in itself infinite, if only for the reason that knowing or understanding the sense of a single term involves, in principle, knowing how to apply it in an infinite number of cases. If, as I have argued, the complex of results running from Cantor, through Gödel, to Tarski, shows the irreducibility of this access to any finitely specifiable procedure, it also thus motivates a realism about sense and its givenness that outstrips any determination of this givenness in terms of (finitely specifiable) capacities, abilities, faculties or practices. Although, as we have seen, this does not by itself demand or establish realism about any particular domain of entities or referents, it is thus the appropriate meta-formal basis for an "ontologically" realist position about sense in its constitutive relation to the being of beings. As applied to the ontological problematic, this realism about the constitution of sense is also thereby an *ontological*

[34] Something similar can be said, as well, about Wittgenstein's own complex and much disputed suggestions in the *RFM* about the Gödel sentence and the meaning and bearing of Gödel's proofs. In particular, though Wittgenstein is certainly dismissively critical of the thought that Gödel, in showing the unprovability of the Gödel sentence, has proven a new mathematical *truth*, this does not mean, as I have argued in the *Politics of Logic*, that what Wittgenstein says cannot be seen as itself suggesting a kind of realism with respect to mathematical *problems* (essentially, on the paradoxico-critical rather than the generic side of the distinction between the two post-Cantorian positions). For instance, in a helpful reading of these remarks, Floyd and Putnam (2000) suggest that Wittgenstein can be seen as anticipating the thought that Gödel's result may show (only) that there is no model of ZFC that includes (only) the natural numbers. Though this suggests that Gödel's result does not after all establish any truths about a "realm" of the natural numbers themselves supposedly given in advance, it is nevertheless a telling meta-formal result *about models*, and one that must apparently have a "realist" construal if its genuine significance is to come into view. Cf. Livingston (2012), chapter 6 (esp. pp. 155-160).

realism: one capable, that is, of maintaining a "realist" attitude not only with respect to beings but to being itself. The significance of this realism is that it must be possible to maintain bivalence *even with respect to 'ontological' discourse*: that is, even the statements which articulate the ultimate basis for the givenness or accessibility of sense in its ontic-ontological structure must themselves be subject to bivalence, bearing determinate truth values independently of "our" ways of knowing them, accessing them, or rendering them "intelligible."

The presupposition of realism in this sense is, of course, essential to the demonstration of results such as Gödel's own incompleteness theorems, and it is equally essential to each of the two horns of the disjunctive conclusion he draws in 1951.[35] It is also a necessary formal precondition for the demonstration of the meta-logical paradoxes (such as Russell's paradox) which condition the post-Cantorian orientations: even if these are indeed taken to show specific points of the violation of the law of *non-contradiction*, it is essential to their positive demonstration that the law of the excluded middle and bivalence (i.e. the claim that each meta-mathematical statement is *either* true or false (or indeed, perhaps both)) be maintained. The present suggestion is just that this sort of realism should *also* be extended to the "ontological" statements which specify the relationship between "being" and "beings as a whole" as specifications of the ontological difference. The result is a rigorously founded "ontological" realism which allows us realistically to characterize the logical, formal, and ultimately *temporal* structure of the difference itself.

It is in this way, as I shall argue, that an attitude of meta-formal realism can also support an ontologically articulated realism about "given" time. That time is "given", in the sense relevant to this investigation, means not only that it is experienced or experienceable by a subject of consciousness but *also*, equally and crucially, that it is *thinkable* in general, and so that thought comprehends temporal determination not only with respect to its condition of presentation or representation within an individual subjectivity but also *in general* and outside this constraint. As I shall argue in succeeding chapters, it is under such a twofold condition – both as given to experience and in thought – that the structure of given time must ultimately be understood by the ontological problematic. In its "metaphysical" determination in onto-theology and constructivism, this twofold givenness is thought as the duality of a finite temporal condition of sensible subjectivity, on one hand, and an atemporal or timeless realm of the intelligible, on the other. However, the development of the ontological problematic beyond the metaphysics of presence itself demands that the time of experience and the time of thought be subject to a uniform and realist criterion. Under this condition, the assumption of bivalence just amounts to the assumption of the meaningfulness of time-determinations in general, and to the (possible) determinacy of statements involving such determinations in general. For this reason, as we shall see over the next few chapters, it can also be the basis of a robust realism about *time* in its fundamental structure, in opposition to the "metaphysical," ultimately anthropological intuition, running from Aristotle to Kant, that locates its basis in the activities, procedures and capacities of a thinking subject of consciousness. Thought this way, the characterization of the metalogical and ontological structural basis for this general

[35] In particular, even if the incompleteness results are seen as indicating the existence of *problems* forever unsolvable, their very objectivity as problems turns on the consideration that their possible answers nevertheless have determinate truth-values.

thinkability of time is moreover the essential requisite for an ontological clarification of the structure of given time.

IV

I have suggested that what I have called meta-formal realism provides a rigorous and appropriate basis for a development of Heidegger's own problematics of sense and time. Besides providing for an underlying realism with respect to these structures and indeed to the question of givenness itself, it relates them to some of the most significant developments of contemporary formal reflection. The question may here arise, though, whether any such application of formal methodology (or methodology developed in accordance with the results and techniques of modern, symbolic logic) can really be made with respect to what Heidegger calls "fundamental ontology" or (later) "the history of Being" at all. For did not Heidegger himself resolutely and repeatedly oppose the application of the "empty" and "merely caulculative" methods of formal, symbolic logic or "logistics" to the question of Being itself? As I have noted, my attempt in this book is not primarily to develop an exegetically faithful reading of Heidegger, but rather to contribute to the development of several interrelated problems that he first pointed out, so it is a matter of relative indifference whether the specific kind of position that I have summarized as metaformal realism can indeed be attributed to Heidegger himself. Nevertheless, it is worth briefly considering the substance of his critique of the application of formal methods to ontology in order to more completely specify the underlying problematics.

It is certainly true that Heidegger often, and throughout his career, opposes any conception according to which the techniques and methods of formal/symbolic logic, for instance of the kind developed by Frege, Russell and Whitehead, can by themselves determine ontological questions or clarify ontological problems. Already in the very early 1912 article "Recent research in logic," for example, Heidegger suggests that calculative "logistics" of the sort developed by Russell in *The Principles of Mathematics* is characterized by inherent "limits" in that it tends to "conceal the meanings of concepts and their shifts in meaning," thus leaving "the deeper sense of principles…in the dark".[36] Logistics in this sense, according to Heidegger, is "simply not familiar with the problems of the theory of judgment" and its "mathematical treatment of logical problems" thus reaches "limits at which [its] concepts and methods fail, more precisely, there where the conditions of [its] possibility lie."[37] In Heidegger's subsequent work, the dominance of logistics (sometimes identified or associated with "positivism") and its substitution for "true" logic is often seen as, more broadly, representative of a broader regime of "calculative thinking" which is characteristic of the contemporary epoch of technology and its privileging of the real in the sense of "actuality" [*Wirklichkeit*]. A passage from the 1941 text "Recollection in Metaphysics" may be considered typical of this:

[36] GA 1, p. 42.
[37] GA 1, p. 42. Nevertheless, in the article Heidegger praises Frege's work, especially in "On Sense and Reference" and "On Concept and Object" as "not yet appreciated in their true significance, let alone exhausted," and as essential not only for "any philosophy of mathematics" but also for "a universal theory of the concept." (GA 1, p. 20).

The precedence of what is real [der Vorrang des *Wirklichen*] furthers the oblivion of Being [betreibt die Vergessenheit des Seins]. Through this precedence, the essential relation to Being which is to be sought in properly conceived thinking is buried. In being claimed by beings [in der Beanspuchung durch das Seiende], man takes on the role of the authoritative [maβgebende] being. As the relation to beings, that knowing suffices [genügt das Erkennen] which, according to the essential manner of beings [Wensensart des Seienden] in the sense of the planned and secured real [des planbar gesicherten Wirklichen] must issue into objectification and thus to calculation [in der Vergegenständlichung aufgehen und so zum Rechnen werden muβ]. The sign of the degradation of thinking [Herabsetzung des Denkens] is the elevation of logistics [Hinaufsetzung der Logistik] to the rank of the true logic. Logistics is the calculable [rechenhafte] organization of the unconditional lack of knowledge [der unbedingten Unwissenheit] about the essence of thinking, provided that thinking, essentially thought, is that projecting knowledge which issues from Being in the preservation of truth's essence [das in der Bewahrung des Wesens der Wahrheit aus dem Sein aufgeht].[38]

Heidegger thus connects the "elevation" of logistics in the sense of calculation to the status of a "true logic" with the more general "precedence" of the real which involves a conception or interpretation of all that is real in being in terms of its capacity to act on and affect beings. This regime is prepared, according to Heidegger, from long ago by the metaphysical interpretation of Being in terms of beings and by the privileging of "thatness", "reality," or "actuality" as the basic character of beings. Within this interpretation, Heidegger suggests, the techniques of mathematical "calculation" or "construction" attain the significance of demonstrating the existence of "something effective within a context of calculative proof." [39] These techniques of calculation and construction thus become the basis for the constitution of the idea of effective causality that underlies "modern" physics and technology and thereby comes to dominate the knowledge and practices of the modern age. With this dominance of the actual in the sense of causally acting and effecting, the "essential determination" of the history of Being is "carried out to its prefigured completion."[40]

[38] GA 6, p. 445. Transl. modified.
[39] GA 6, p. 419.

[40] GA 6, p. 419. The passage in full (from "Metaphysics as the History of Being"): "The usual name for thatness [das Daβ-sein], existence, testifies to the precedence of Being as *actualitas* in this interpretation. The dominance of its essence as *reality [Wirklichkeit]* determines the progression of the history of Being, throughout which the essential determination once begun is carried out to its prefigured completion. The real is the existing. [Das Wiirkliche ist das Existierende]. This includes everything which through some manner of causality [Verursachung] *constituitur extra causas*. But because the whole of beings is the effected and effecting product of a first producer [die Gewirkte-Wirkende eines ersten Wirkers ist], an appropriate structure enters the whole of beings [kommt in die Ganze des Seienden ein eigenes Gefüge] which determines itself as the co-responding of the actual produced being [des jeweilig Gewirktem] to the producer [zum Wirker] as the highest being. The reality [Wirklichkeit] of the grain of sand, of plants, animals, men, numbers, corresponds to the making of the first maker [entspricht dem Wirken des ersten Wirkers]. It is at the same time like and unlike his reality [Wirklichkeit]. The thing which can be experienced and grasped with the senses [handgreifliche] is existent, but so is the object of mathematics which is nonsensuous and calculable [der nichtsinnliche errechbare Gegenstand der Mathematik]. "M exists" means: this quantity can be unequivocally constructed [ist … eindeutig konstruierbar] from an established point of departure

Heidegger thus sees the calculative techniques of symbolic and mathematical logic as, on the one hand, "empty" with respect to the actual structure and nature of presence and presencing themselves and, on the other, symptomatic in their growing dominance of the "metaphysical" conception of Being in terms of beings as it moves toward completion. The position is in a certain way overdetermined with respect to the actual "content" of the techniques of mathematical logic themselves: though these techniques are in themselves empty and incapable of supporting "thinking, essentially thought", nevertheless their contemporary dominance, in connection with the regime of technology that they make possible, points in an important and even privileged way to what is most preeminently to be thought today. Despite this air of overdetermination, though, one might easily conclude from what Heidegger says that no methodology or result that essentially depends on formal or mathematical logic can play any positive role in furthering the ontological problematic itself, either in the sense of the "fundamental ontology" of Dasein or in the later sense of the history of being.

The methodology of meta-formal reflection that I have discussed, and which is modeled by Gödel's reasoning about the implications of his own results, does in fact depend essentially and in an obvious sense on the techniques of symbolic logic and mathematical proof; and so it might be thought, along these lines, that it just cannot be applied to the ontological problematics with which Heidegger is concerned. But in fact, none of the considerations that Heidegger introduces bear in any substantive way against the application of metaformal reasoning that I have suggested here.

First, as we have seen, what is in view with the kind of metaformal reasoning that I have discussed is not at all simply the mechanical application of a "formal" technique of symbol-manipulation, but rather a reflective illumination of the very conditions under which any such logical technique is possible and gains any possible relationship with truth. This reflective illumination, as we saw also in connection with the twofold consideration of truth and meaning in chapter 3, may more closely be compared to the task of what was traditionally called "transcendental" (rather than formal) logic in its evincing of the structure of the givenness of things themselves. But second, and more importantly, far from simply applying an effective technique of empty calculation that is assumed to have universal scope in itself, the "limitative" results of Gödel and Turing point exactly to the formally inherent limits of the actual effectiveness of any such technique. As such, they are themselves formally diagnostic of the configuration of thought and practice that simply *assumes* in advance the unlimited applicability of calculative techniques. Indeed, by demonstrating the necessary existence of the undecidable, the uncalculable, and the ineffective that accompanies *any* formal definition of technical or regular effectiveness, they also provide formally motivated terms for the fundamental critique of this configuration. In reference to Heidegger's 1941 statement about the way that the "elevation of logistics" corresponds to the assumption of the unlimited calculability of beings, it is particularly significant that Turing's result in 1937 demonstrates the existence and ubiquity of (uncountably many)

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of calculation with established methods of calculation [mit festgelegten Rechnungsmitteln]. What is thus constructed [Das so Konstruierte] is thus proven as something effective within a context of calculative proof [als das innerhalb eines Begründungszusammenhanges der Rechnung Wirksame]. "M" is something with which one can calculate [womit man rechnen kann], and under certain conditions must calculate. Mathematical construction [Die mathematische Konstruktion] is a kind of constitution of the *constituere extra causas,* of causal effecting [des verursachended Erwirkens.] (GA 6, p. 419, transl. slightly modified).

real numbers that are *uncomputable* in a precise sense: that is, numbers that are wholly determinate but which cannot be determined by *any* finite procedure of calculation. More generally, these result of formally based reflection on formal methods – whereby these methods are inherently limited, in their relationship to truth, by an essential *ineffectivity* that necessarily accompanies them wherever they are applied – is anticipated in detail (as we have seen in chapter 1, above) by Frege's own conception, in opposition to the dominant psychologism, of logic as the site of an insistence of what is (precisely) real without being actual in the sense of "effective." But the inherent ineffectivity accompanying any total or calculative regime of thinking is only really rigorously demonstrated and positively verified, as we have seen, by the paradoxical and limitative results (including Russell's paradox, Gödel's theorems, and Turing's argument) that follow in quick succession from the completion of the "foundationalist" project itself.

In this respect, again, far from being opposed to Heidegger's consideration of the role of the dominance of "calculative" thought and its assumption of unrestricted applicability in the history of Being, the metaformal results of Gödel and Turing in fact confirm Heidegger's critique and point in a formally rigorous way to the very "closure" of the metaphysical regime of "actuality" that Heidegger himself attempts to describe. Here, it is thus not necessary to oppose the thinking that emerges from reflection on the scope and limits of formal/symbolic logic to the Heideggerian ontological problematic; rather, given the specific positive character of the limitative results that arise from this reflection, they can be seen as directly contributing to the development of this problematic and even confirming it by other means. Heidegger's own animadversions against the usefulness of symbolic logic (or the assumption of its unlimited applicability) are thus no reason to reject the application of metaformal reasoning I have suggested here. Aside from this, though, are there any positive arguments to be found in Heidegger's *corpus* that suffice to establish that formal reasoning of a "logical" or "mathematical" character *cannot* shed light on phenomenological or ontological issues?

By contrast with statements simply asserting the "emptiness" of formal/symbolic logic, or genealogical/historical descriptions of what Heidegger sees as the role of "logic" as such (and primarily in its Aristotelian or Hegelian forms) in the development and fixation of the metaphysical tradition, such positive arguments are much harder to find in Heidegger's texts. One such, however, is suggested in the course of a critical discussion of Husserl's phenomenology in Heidegger's (early) Freiburg lecture course, "Ontology: The Hermeneutics of Facticity," from the summer of 1923. Here, Heidegger challenges what he sees as Husserl's presupposition of "mathematics and the mathematical natural sciences" as a *model* "for all sciences," which according to Heidegger suggested, in the earlier development of Husserl's phenomenology, that phenomenological description itself be "[elevated]…to the level of mathematical rigor:"[41]

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[41] This attribution of this position to Husserl is in fact puzzling in at least two ways. First, of course, given Husserl's longstanding and decisive critique of naturalism and the natural attitude, it can hardly be said (whatever the role of mathematics itself in serving as a model for phenomenological description) that he *generally* privileged "mathematical natural science" as a model for phenomenological investigation. But second, although it is indeed suggested in the *Logical Investigations* that mathematics in the sense of a "mathesis universalis" can serve as a formal structure for all *logical* theory, by 1923 Husserl had already clearly rejected the idea that the

Nothing more needs to be said here about this absolutizing [of mathematical rigor]. This is not the first time it has surfaced, but rather it has for a long time dominated [beherrscht] science, finding an apparent justification in the general idea of science as it appeared among the Greeks, where one believed that knowledge was to be found as knowledge of the universal [das Allgemein] and - what is seen to be the same thing - knowledge of what is universally valid [des Allgemeingültigen]. But this is all a mistake. And when one cannot attain such mathematical rigor, one gives up.

> Fundamentally, one does not even realize that a prejudice lies here. Is it justified to hold up mathematics as a model for all scientific disciplines? Or are the basic relations between mathematics and the other disciplines not thereby stood on their heads? Mathematics is the least rigorous of disciplines, for the access is here the easiest [der Zugang ist hier der allerleichteste]. The human sciences [Geisteswissenschaft] presuppose much more scientific existence than could ever be achieved by a mathematician. One should approach a scientific discipline not as a system of propositions and grounds for justifying them [Begründungszusammenhängen], but rather as something in which factical Dasein critically confronts itself and explicates itself [mit sich selbst auseinandersetzt]. This pre-establishment of a model [Einsetzung eines Vorbildes] is unphenomenological- the meaning of scientific rigor needs rather to be drawn [zu erheben] from the kind of object [being investigated] [aus der Gegenstandart] and the mode of access appropriate to it [der ihr angemssenen Zugangsart].[42]

According to this argument, in other words, it is inappropriate to treat mathematics as the "model" for the phenomenological description of what is given in experience, or methodologically to impose the kind of rigor that is characteristic of it here. This is because, as Heidegger argues, phenomenology is not a topical area or a categorical field but rather a method of developing the "how" of access into what is present in intuition, just as it gives itself to experience there. Since it is concerned with the mode of access in this way, phenomenological description has to be developed according to the kind of access that is characteristic of the particular field or kind of object being investigated in each case, and it is accordingly a mistake to take the characteristic universality and universal transmissibility of mathematical knowledge as a methodological or thematic model for all "scientific" inquiry. In this respect, in fact, Heidegger suggests, this characteristic universality and accessibility of mathematics makes it in fact the "least rigorous" of disciplines, in that it means that it fails to involve the complexity or singularity of the "scientific existence" that the human sciences themselves presuppose and attempt to theorize.

From the perspective of meta-formal reflection that I have suggested here, it should be said, first, that there is no need to presuppose the purported "universality" and accessibility of mathematical *objects* in order to apply the lessons of "metalogical" or "metamathematical" reflection to the problems of (phenomenological) access and givenness. As we have seen, the attitude of meta-formal realism should,

phenomenological structure of experience itself could always be mathematized in a formally exact way: see, e.g. Ideas I (1913) section .

[42] GA 63, p. 72. Transl. slightly modified.

on the one hand, be sharply distinguished from the (vulgar) “Platonist” attitude of assuming or presupposing the timeless existence of a range of mathematical objects universally accessible due to their privileged residence in a kind of *topos ouranous* quite alien to anything specifically involved in “our” form of life; while, on the other, the positive results on which meta-formal realism turns provide grounds for a formally based reconsideration of what is involved – in the theory of proof, the force of rules of inference, and the provision of axioms themselves – in anything that can reasonably be seen as the “accessibility” of mathematical “objects” to begin with.

Second, though, and along the same lines, though, it should also be asked what kinds of accessibility *do* characterize mathematical knowledge, and what is the form underlying these kinds of accessibility in the facticity of a life, here determined not simply in terms of any factual-anthropological conception of the “human” but in a way structurally corresponding to its proper modes of givenness and presence themselves. For mathematics is after all, among other things, an activity undertaken in the course of such a life among other activities of theoretical reflection and practice; and without yet assuming anything determinate about the ontological mode of existence of its objects, it is certain that the problem of access here raises quite specific and difficult problems which must be confronted by *any* phenomenological or ontological theory of givenness or presence as such. Especially in connection with the idea of the infinite, which receives (as we have seen) a fundamental and transformative articulation in the work of Cantor and the developments which follow him, these are problems of “access” that are not in fact limited to the “philosophy of mathematics” in a narrow sense, but rather raise questions bearing on the structural form of “our” mode of life (for instance, the nature and meaning of its long-discussed “finitude”) itself. As I have tried to argue here, there are also not distinct from the problems constitutively involved in any account of “our” access to *meaning* or *sense* and indeed of its own basic constitution, insofar as this basic constitution *always* involves the “infinite” character of the one over the (unlimited) many. These are the problems visibly taken up in an original fashion (although not resolved) by Plato in the heroic dialectics of his late attempts at a revision of the classical “theory of forms;” and, as I have tried to show (especially chapter 2) they are also problems that can by no means be avoided by an ontological hermeneutics in its own development of the question of access and accessibility, most of all where this question overlaps with the problem of truth. Here, indeed, as I shall attempt to demonstrate over the next several chapters, the insistence of these problems points in a basic structural way to the original problem of the givenness of time, insofar as it can be experienced or measured at all.

For these reasons, over the next several chapters of the investigation, we shall take up the old problem of “mathematical existence” on a renewed ontological-hermeneutic ground, not with a view to establishing or securing the model of mathematical objectivity as absolute existence, but as the concrete problem of the form of sense as it communicates with the structure of temporality and with presence “in general.” The investigation will lead us to consider such matters as the possible “givenness” of the infinite to thought, the peculiar temporal character of “historical” languages which are nevertheless capable of expressing judgments and truths “once and for all,” and the mysterious thought of a superior “ideal genesis” of forms that Plato appears to assay, in his last writings and unwritten doctrines, on the model of an actual origin of numbers from the superior principles of the one and the unlimited many themselves. The aim will be a substantial clarification, relevant to the contemporary "ontological" situation, of the problem of the sense and truth of Being insofar as it comes to light as time.